In this exercise we will simulate motion of a system resembling the eye. Our objective is to find the motor commands that minimize endpoint variance.

$$A = \begin{bmatrix} 1 & 0.001 & 0 \\ -0.34 & 0.92 & 0.34 \\ 0 & 0 & 0.9 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 0.1 \end{bmatrix}$$

$$\mathbf{x}^{(n)} = A\mathbf{x}^{(n-1)} + \mathbf{b}\left(u^{(n-1)} + \varepsilon^{(n-1)}\right) + \eta^{(n-1)}$$

$$\varepsilon^{(i)} \sim N\left(0, \kappa^2 \left(u^{(i)}\right)^2\right)$$

$$\eta^{(i)} \sim N\left(0, \sigma_\eta^2\right)$$

The step size is 0.001 sec.
We wish to minimize variance at time step *p*, and then maintain the eye at the target *g* for duration *k*.

$$J = \mathrm{var}\left[\mathbf{s}^T \mathbf{x}^{(p)}\right]$$

$$\mathbf{s}^T E\left[\mathbf{x}^{(p+i-1)}\right] = g \quad \text{for } 1 \le i \le k$$

$$\mathbf{s} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

The augmented cost to minimize is as follows:

$$J = \mathrm{var}\left[\mathbf{s}^T \mathbf{x}^{(p)}\right] + \sum_{i=1}^{k} \lambda_i \left(\mathbf{s}^T E\left[\mathbf{x}^{(p+i-1)}\right] - g\right)$$

$$E\left[\mathbf{x}^{(m)}\right] = A^m \mathbf{x}^{(0)} + \sum_{j=0}^{m-1} A^{m-1-j} \mathbf{b} u^{(j)}$$

$$J = \mathrm{var}\left[\mathbf{s}^T \mathbf{x}^{(p)}\right] + \sum_{i=1}^{k} \lambda_i \left(\mathbf{s}^T A^{p+i-1} \mathbf{x}^{(0)} - g + \sum_{j=0}^{p+i-2} \mathbf{s}^T A^{p+i-2-j} \mathbf{b} u^{(j)} \right)$$

The starting point is: $\mathbf{x}^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ Target is at: $g = \dfrac{\pi}{5}$

p is 120 (i.e., 0.120 sec) (this is the duration of the movement), and hold time *k* is 50 (i.e., 0.050sec) (this is the duration of the period post movement completion).

The state vector represents position, velocity, and force: $\mathbf{x} = \begin{bmatrix} x \\ \dot{x} \\ f \end{bmatrix}$

Assuming no signal dependent noise, i.e., $\kappa = 0$
compute the optimal motor commands and plot it. Also plot the resulting position and velocity (in deg and deg/sec, rather than in radians).

Now repeat the process using signal dependent noise $\kappa = 0.00006$
Compute the optimal motor commands and plot it. Also plot the resulting position and velocity.

Note that presence of non-signal dependent noise has not bearing on the solution.