

$$\underline{df} \equiv \begin{bmatrix} df_x \\ df_y \end{bmatrix} \quad \underline{dx} \equiv \begin{bmatrix} dx_x \\ dx_y \end{bmatrix}$$

Data set:  $\{\underline{dx}, \underline{df}\}_1, \{\underline{dx}, \underline{df}\}_2, \dots, \{\underline{dx}, \underline{df}\}_n$

Objective: find the best fit to  $K_x = \frac{df}{dx}$  where  $K_x = \begin{bmatrix} K_{x11} & K_{x12} \\ K_{x21} & K_{x22} \end{bmatrix}$

$$\underline{df} = K_x \underline{dx}$$

$$\text{error in fit for data set 1: } \varepsilon \equiv \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \end{bmatrix} = \begin{bmatrix} df_x - K_{x11} dx_x - K_{x12} dx_y \\ df_y - K_{x21} dx_x - K_{x22} dx_y \end{bmatrix}$$

$$\text{cost function to minimize: } e \equiv \varepsilon^T \varepsilon = (df_x - K_{x11} dx_x - K_{x12} dx_y)^2 + (df_y - K_{x21} dx_x - K_{x22} dx_y)^2$$

$$\frac{de}{dK_{x11}} = -2(df_x - K_{x11} dx_x - K_{x12} dx_y) dx_x = -2\varepsilon_x dx_x$$

$$\text{Gradient descent: } (K_{x11})_{n+1} = (K_{x11})_n - \eta \frac{de}{dK_{x11}} = (K_{x11})_n + 2\eta \varepsilon_x dx_x$$

## Arm's stiffness in joint coordinates

$$J(\underline{q}) = \frac{d\underline{x}}{d\underline{q}}$$

$$K_x = \frac{d\underline{f}}{d\underline{x}} \quad K_j = \frac{d\underline{\tau}}{d\underline{q}}$$

$$\underline{\tau} = J(\underline{q})^T \underline{f}$$

$$K_j = \frac{d(J(\underline{q})^T \underline{f})}{d\underline{q}} = \frac{d(J(\underline{q})^T)}{d\underline{q}} \underline{f} + J(\underline{q})^T \frac{d\underline{f}}{d\underline{q}}$$

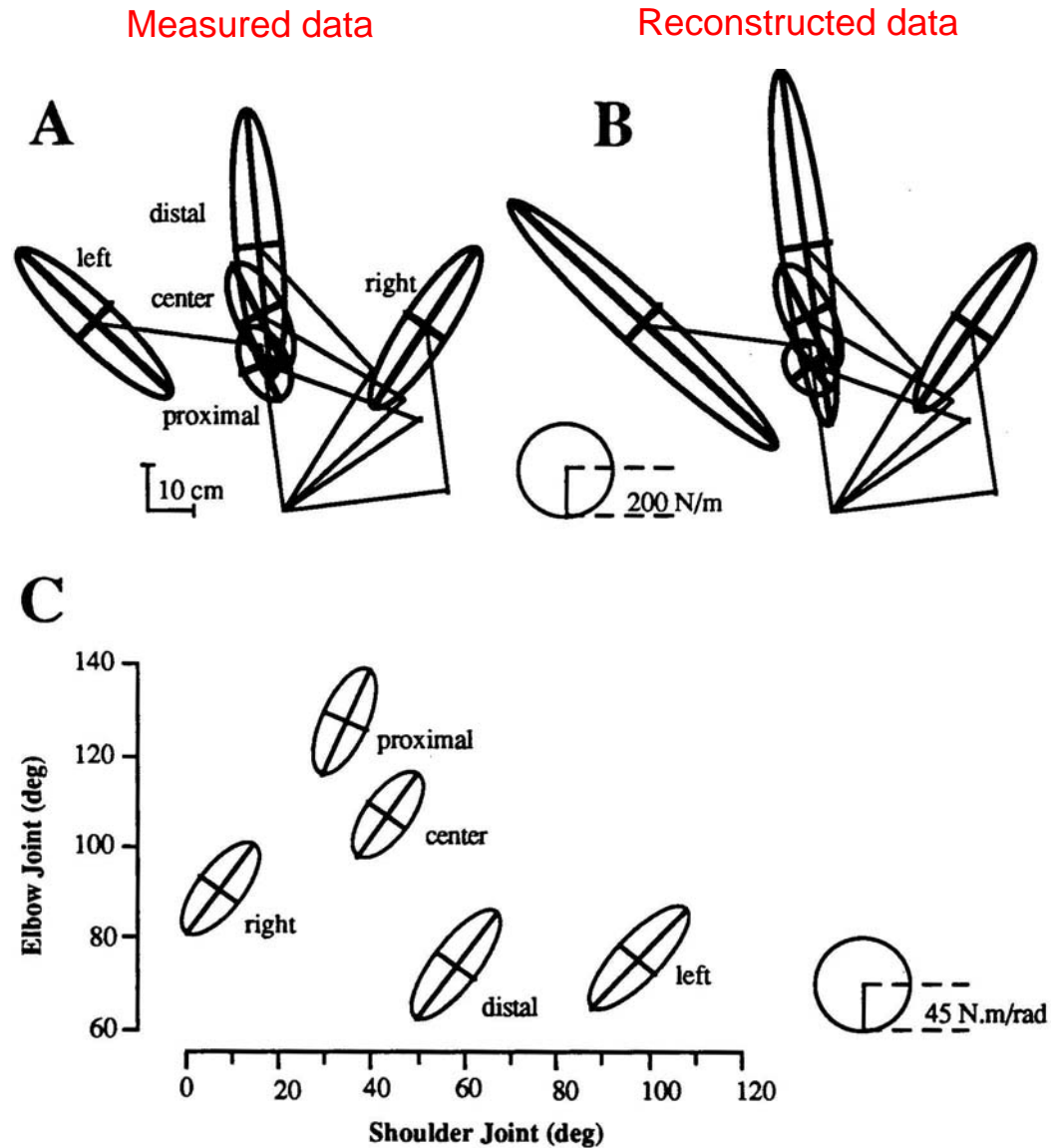
for small displacements, assume  $\frac{d(J(\underline{q})^T)}{d\underline{q}} \approx 0$

$$K_j = J(\underline{q})^T \frac{d\underline{f}}{d\underline{q}} = J(\underline{q})^T \frac{d\underline{f}}{d\underline{x}} J(\underline{q})$$

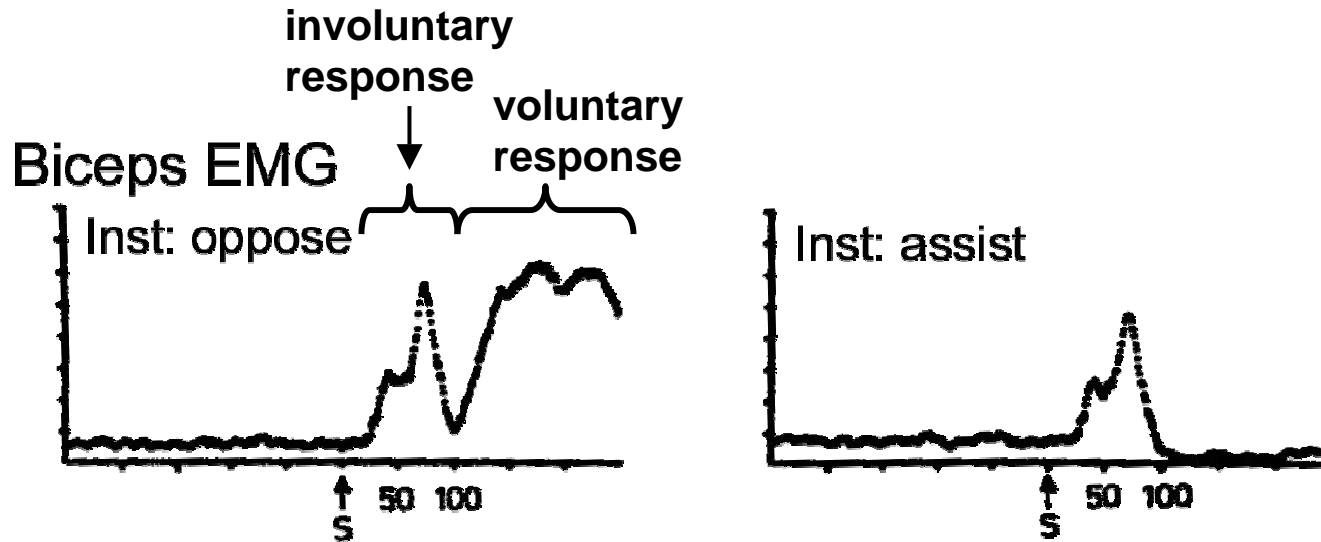
$$K_j = J(\underline{q})^T K_x J(\underline{q})$$

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There is remarkable regularity in the stiffness of the arm when it is expressed in joint coordinates.  $K_j$  is essentially constant and does not vary as a function of arm position. The reconstructed data is from the  $K_j$  measured at the center position.



## Time delays in the short- and long-loop reflexes



Torque perturbations were imposed on the elbow of a subject at random times (S indicates the time of perturbation). On each trial, an LED instructed the subject to either oppose the torque or assist the torque.

# Instruction dependent activity in the cerebellar dentate nucleus in response to a perturbation

